A Cascade Theory of Drag Reduction

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A Cascade Theory of Drag Reduction.

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Abstract. – We describe homogeneous, isotropic, three-dimensional turbulence in a dilute solution of neutral, flexible chains. Energy flows by the usual cascade down to a scale r^* such that the shear rates $U(r^*)/r^*$ (U being the velocity) become equal to the relaxation rate of one coil. At small scales r, the molecules follow affinely the deformation of a local volume element. At a certain smaller scale r^{**} the elastic stresses in the coils become comparable to the Reynolds stresses. The polymer truncates the cascade when r^{**} becomes larger than the usual Kolmogorov limit r_k. This defines a critical concentration, which depends on *both* polymer and flow parameters.

Very dilute polymers (of concentration $c \leqslant 100$ p.p.m.) cause a significant drag reduction in turbulent flow [1]. Recent experiments with polymer injected far from the walls [2] suggest that the effect need not to be connected with the laminar boundary layer. This observation led us to some tentative thoughts on homogeneous, isotropic, turbulence at high Reynolds numbers in a polymer solution, to be described below.

1. A related laminar problem.

Our starting point is a discussion of a converging, laminar flow at the entry of a capillary [3, 4] (see fig. 1). At large distances r from the entrance point, the shear rates are small and the polymer coils retain their equilibrium shape, while drifting towards the entry. At a certain distance r^* the local shear rate becomes equal to the Zimm relaxation rate of the coils $1/\tau$. From this moment on, the chains must follow the ambient fluid without adjustment: they are affinely deformed. For instance, in 3-dimensional converging flow, the dimensionless elongation λ of the coils is $\lambda = (r^*/r)^2$, while in a 2-dimensional case, $\lambda = r^*/r$. Ultimately the coil reaches the capillary, and enters if its lateral size is smaller than the capillary diameter [3].

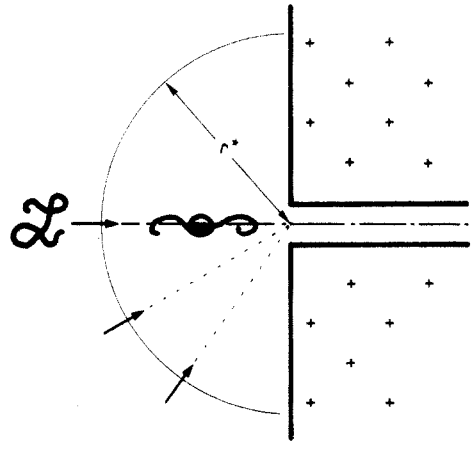

Fig. 1.

2. The trapping length r^*.

Let us transpose these ideas to 3-dimensional turbulence, following the Kolmogorov picture [5]: we start with a prescribed energy dissipated per unit mass ε; at large scales r (Small shear rates) the coils are undistorted, and the velocity $U(r)$ follows

$$\varepsilon = \frac{U^3(r)}{r} . \tag{1}$$

At a certain scale r^* the shear rate becomes equal to τ^{-1}

$$\frac{U(r^*)}{r^*} = \frac{1}{\tau} . \tag{2}$$

Thus $r^* = \varepsilon^{1/2} \tau^{3/2}$ is independent of polymer concentration, but dependent on both τ and ε.

3. The affine deformation regime.

Here again we expect to have a relation between the elongation λ and the scale r, of the general form

$$\lambda = \left(\frac{r^*}{r}\right)^n \qquad (r < r^*) , \tag{3}$$

where n is an exponent which depends on the detailed statistics of the deformation matrix ($2 \geqslant n$). The molecules become elongated, but, for a certain range of spatial scales, their reaction on the flow is still negligible. Thus we retain the Kolmogorov equation (1) at scales $r < r^*$.

4. The elastic limit.

If each coil behaves like small spring, and the deformation λ implies an elastic energy (per cm^3)

$$\frac{1}{2} G(\lambda - 1)^2 \sim \frac{1}{2} G\lambda^2 \qquad (\lambda \gg 1) .$$

The elastic modulus is $G \simeq \nu\, kT$ (ν being the number of coils/cm^3 and kT the thermal energy) ([1]). The flow behaviour is strongly modified when this elastic energy becomes equal to the kinetic energy. This defines a second characteristic length r^{**}, such that

$$\frac{1}{2}\rho U^2(r^{**}) = \frac{1}{2} G\lambda^2(r^{**}) \,, \tag{4}$$

ρ being the density of the fluid. Equivalently, eq. (4) may be considered as a balance between Reynolds stresses and elastic stresses. It is convenient to introduce a characteristic velocity $s = (G/\rho)^{-1/2}$ and a dimensionless ratio $\psi = s/U(r^*)$. Then eq. (4) can be rewritten as

$$\frac{r^{**}}{r^*} = \psi^{\alpha} \,, \qquad \alpha = \frac{3}{1+3n} \,. \tag{5}$$

In the small concentration regime of interest, ψ is small, and $r^{**} \ll r^*$. Elongation cannot proceed further than the limit $\lambda(r^{**})$: turbulence is suppressed at small scales; each volume element develops large tensions, as in a tubeless siphon [6].

5. Comparison of limits.

In the absence of polymer, the energy cascade went down to a minimal scale r_k controlled by viscous dissipation (5)

$$r_k = r^*(\mathrm{Re}^*)^{-3/4}, \tag{6}$$

where Re* is the Reynolds number of the scale r^*

$$\mathrm{Re}^* = \frac{pU(r^*)\, r^*}{\eta} \,.$$

The presence of the polymer will alter the cascade if, and only if, the elastic cut off r^{**} becomes larger than the Kolmogorov cut-off r_k. Thus, for a given flow, and a given choice of polymer (defining r^*) there is a critical concentration c_0, or an equivalent critical $\psi = \psi_0$, such that $r^{**}(\psi_0) = r_k$. This gives

$$\psi > \psi_0 = (\mathrm{Re}^*)^{-3/4\alpha} \,, \tag{7}$$

ψ_0 and c_0 depend on both flow parameters and polymer parameters in an intricate way. One useful presentation of eq. (7) is in terms of the (low shear) added viscosity due to the polymer $\delta\eta \simeq \nu\tau\, kT$

$$\rho^{-1}\delta\eta_0 = \mu^m \left(\frac{r^{*2}}{\tau}\right)^{1-m} \,, \tag{8}$$

where μ is the kinematic viscosity of the solvent, and $m = 3/2\alpha$.

([1]) For coils in good solvents, at $\lambda \gg 1$, the energy is $\sim \lambda^{5/2}$ rather than λ^2: see PINCUS, *Macromolécules*, **9** (1976) 386. We ignore these refinements for the moment.

6. Discussion.

The conventional «time criterion» [1] for the Toms effect amounts to assuming that some shear rates in the cascade are faster than $1/\tau$, *i.e.* that there does exist a scale r^*. This is a necessary, but not sufficient condition for polymer effects: the crucial condition is eq. (7) or (8). When the concentration is beyond threshold, the Kolmogorov cascade is truncated. Studies of drag reduction in turbulent, free shear, flows (for which the cascade model is rather plausible) all seem to demonstrate a clear suppression of small scales [7, 8]. For turbulent pipe flow, the relation to cascade theory is more complex. We can calculate a limit $r^{**}(y)$ at all distances y from the wall; the structure of this limit is very different from the cut-off of Lumley [1], based on a pure renormalization of viscosity. But the result is also a thickening of the buffer layer.

Note finally that we have decribed only one «scenario» for the cascade. Some others can exist: for instance eq. (3) may break down at a point where the chains are fully elongated. In the first scenario described by eqs. (3), (4), the essential feature is the *elastic* behaviour of coils at high frequencies.

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